Theory of pinning in a Superconducting Thin Film Pierced by a Ferromagnetic Columnar Defect

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This is an analytical study of pinning and spontaneous vortex phase is a system consisting of a superconducting thin film pierced by a long ferromagnetic columnar defect of finite radius R. The magnetic fields, screening currents, energy and pinning forces for this system are calculated. The interaction between the magnetic field of vortices and the magnetization outside the plane of the film and its close proximity enhances vortex pinning significantly. Spontaneous vortex phase appears when the magnetization of the columnar defect is increased above a critical value. Transitions between phases characterized by different number of flux quanta are also studied. These results are generalized to the case when the superconductor is pierced by an array of columnar defects.

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I. INTRODUCTION

Optimizing pinning in superconductors (SC) is a problem that is interesting both experimentally and theoretically since pinning is the main viable mechanism for superconductivity in the presence of external magnetic field. Many tools and mechanisms for pinning has already been studied, in particular the use of crystal defects such as holes, non-magnetic impurities and both linear and screw dislocations. Pinning using structural defects suffers from many drawbacks, the most important is that the pins are randomly distributed which results in a low critical current. In recent years, it was claimed that pinning could be optimized if we employ ferromagnetic (FM) nano textures to pin superconducting vortices (SV).[1]-[13]

The main obstacles of using ferromagnetic textures to pin superconducting vortices are the proximity effects which destroy superconductivity. Lyuksyutov and Pokrovsky noticed that proximity effects could be suppressed if a thin layer of insulator oxide is sandwiched between the FM and the SC.[5] Due to this separation between the FM and SC, the interaction between them is mediated via their magnetic fields. If the magnetization of the FM structure exceeds a threshold value M_c, the interaction between the FM and SC makes the spontaneous creation of superconducting vortices energy favorable. The interaction between the FM and SC increases as the temperature reaches the superconducting transition temperature T_s from below.

Marmorkos *et.al.*[8] considered a system consisting of a thin ferromagnetic dot embedded in a superconducting thin film. Both the superconductor and the ferromagnet are assumed to lie in the xy-plane and the dot is magnetized along z-axis. By numerically solving the nonlinear Ginzburg-Landau equation, they were able to show that a vortex appears in the superconductor when the magnetization of the dot exceeds a critical value. They also showed that increasing the dot's magnetization leads to a giant vortex state with multiple flux quanta.

In Ref.7, Erdin *et.al.* used London's theory of superconductors to solve the general problem of the interaction between vortices in superconducting thin film with a generic two-dimensional ferromagnetic structure. They applied their results to the cases when the FM structure is a circular dot whose magnetization is either parallel or perpendicular to the plane of the superconductor. They calculated the threshold value for the dot's magnetization at which vortices are spontaneously created in the superconductor. They showed that by increasing the magnetization of the dot a series of phase transitions between phases with different number of vortices take place.

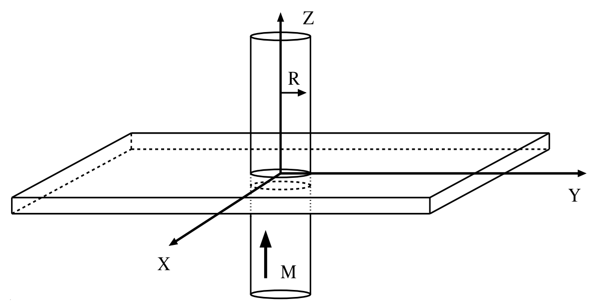

FIG. 1: A superconducting thin film pierced by a ferromagnetic nano rod of radius R, length $2L$ and magnetization M.

All these studies focused on cases in which the ferromagnet is either a point-like dipole or an infinitely thin two-dimensional texture whose plane is parallel to the plane of the superconductor. The problem of interaction between vortices in superconducting thin film with a ferromagnetic columnar defect (FCD) has not been studied yet. This case is both theoretically and experimentally interesting for the following reason. The magnetic field of the vortex in a superconducting thin film is not confined to the plane of the film but it also exist outside of it. If the ferromagnet extends in space outside the film and its close proximity then the vortex will be bound more strongly to the ferromagnet due to the interaction of its magnetic field with the magnetization of the ferromagnet. This enhances vortex pinning and consequently the critical current of the superconductor. It is also important to note that ferromagnetic columnar defects makes

the superconductor has a multiply connected topology which is another source of pinning. Therefore, it would be interesting both theoretically and experimentally to study the statical and dynamical properties of a superconducting thin film pierced by ferromagnetic columnar defects.

In this article, I propose to study the properties of spontaneous vortex phase and pinning in a superconducting thin film pierced by ferromagnetic columnar defects. This article is organized such as in the first section, the magnetic potential and field distribution produced by a single FCD penetrating a thin superconducting film are calculated. In sections two, I calculate the total energy of a system of a superconducting vortex coupled to the magnetic defect. I generalize these results to the case of a superconducting thin film pierced by a square array of magnetic columnar defects in section four then I summarize the results of this work in section five.

II. AN SC FILM PIERCED BY A FERROMAGNETIC COLUMNAR DEFECT

I consider a superconducting thin film in the xy-plane pierced by a finite ferromagnetic nano rod whose radius is R and length is L as shown in Fig. 1. I also assume that the FCD is uniformly magnetized along its symmetry axis then its magnetization distribution is written as

$$\mathbf{M}(\rho,\varphi,z) = M\Theta(R-\rho)\Theta(\frac{L}{2}-|z|)\hat{z} \tag{1}$$

where $\hat{z}$ is the unit vector along the z-axis. The magnetic field produced by the FCD penetrates the superconductor and changes the distribution of the screening current. In the London approximation the FCD-SC system is described by London-Maxwell equation

$$\nabla\times\nabla\times\mathbf{A} = \frac{4\pi}{c}\mathbf{J} \tag{2}$$

The total current $\mathbf{J} = \mathbf{J}^s + \mathbf{J}^m$ is the sum of the supercurrent $\mathbf{J}^s$ and the magnetization current $\mathbf{J}^m = c\nabla\times\mathbf{M}$. The supercurrent can be written in terms of the gradient of the superconducting order parameter χ and the total magnetic vector potential $\mathbf{A} = \mathbf{A}^s + \mathbf{A}^m$ for the FCD-SC system $\mathbf{J}^s = \frac{n_s\hbar e}{2m_e}\left(\nabla\chi - \frac{2\pi}{\phi_0}\mathbf{A}\right)$. Here n_s is the Superconducting electron density, $\hbar$ is Planck's constant while m_e and e are the electron's mass and charge respectively. The superposition principle allows us to solve (2) for $\mathbf{A}^m$ and $\mathbf{A}^s$ independently. If the Coulomb gauge $\nabla\cdot\mathbf{A}^m = 0$ is imposed then $\mathbf{A}^m$ satisfies the following equation

$$-\nabla^2\mathbf{A}^m = -\frac{1}{\lambda}\mathbf{A}^m\delta(z) + 4\pi\nabla\times\mathbf{M} \tag{3}$$

where $[\lambda = \lambda_L^2/d_s]$ with $[\lambda_L = \sqrt{m_e c^2/4\pi n_s e^2}]$ is London's penetration depth. Here m_e and e are the electron's mass and its charge while n_s is the superconducting electrons density and d_s is the thickness of the SC film. The general solution of (3) is of the form

$$\mathbf{A}^m = \frac{1}{(2\pi)^3}\int \tilde{\mathbf{A}}^m_{\mathbf{k}} e^{-\imath(\mathbf{q}\cdot\rho + k_z z)} d^2q dk_z \tag{4}$$

where k_z and $\mathbf{q}$ are the components of the wave vector along the z-axis and in the xy-plane. Here $\tilde{\mathbf{A}}^m_{\mathbf{k}}$ is the vector potential in momentum space and is given by

$$\tilde{\mathbf{A}}^m_K = \frac{16\pi^2\imath MRJ_1(qR)}{k_z^2+q^2}\left[\frac{2\sinh(\frac{qL}{2})e^{-\frac{qL}{2}}}{q(1+2\lambda q)} - \frac{\sin(\frac{k_z L}{2})}{k_z}\right]\hat{\varphi}_q \tag{5}$$

where $J_n(x)$ is the n-th rank Bessel's function. The magnetic field produced by the magnetic nano-rod in the presence of the SC film can be calculated using $\mathbf{B} = \nabla\times\mathbf{A}^m$. The components of the rod's magnetic field are

$$B_z = \frac{8\pi MR}{\lambda}\int_0^\infty J_1(qR)J_0(q\rho)\left[\frac{\pi}{2q^2}W(q,z,L) - \frac{\sinh(\frac{qL}{2})e^{-q(|z|+\frac{L}{2})}}{1+2q}\right]dq \tag{6}$$

$$B_\rho = \frac{8\pi MR}{\lambda}\int_0^\infty J_1(qR)J_1(q\rho)\left[\frac{\pi}{2q^2}W(q,z,L) - \frac{\sinh(\frac{qL}{2})e^{-q(|z|+\frac{L}{2})}}{1+2q}\right]dq \tag{7}$$

where $W(q,z,L)$ is

$$\begin{aligned} W(q,z,L) &= sign(L-2z)\left[1-\cosh(\frac{q(L-2z)}{2}) + sign(L-2z)\sinh(\frac{q(L-2z)}{2})\right] \\ &+ sign(L+2z)\left[1-\cosh(\frac{q(L+2z)}{2}) + \sinh(\frac{q|L+2z|}{2})\right] \end{aligned} \tag{8}$$

However, we are interested in the value of the field at the plane of the superconductor. The z component of the magnetic field of the FCD evaluated at the SC plane is

$$B_z^m(\rho) = \frac{8\pi MR}{\lambda}\int_0^\infty \frac{qJ_1(qR)J_0(q\rho)}{1+2q}(1-e^{-\frac{qL}{2}})dq \tag{9}$$

Similarly the solution for $\mathbf{A}^s$ is found. A general argument made in[7] shows that the term proportional to $\nabla\chi$ ascribes for vortices. So, in the presence of a vortex with vorticity ν and center at $\rho_0 = 0$, the solution for $\mathbf{A}^s$ in the Coulomb gauge is

$$\mathbf{A}^s(\rho, z) = \frac{\nu\phi_0}{2\pi}\hat{z} \times \hat{\rho} \int_0^\infty \frac{J_1(q|\rho|)e^{-q|z|}}{1 + 2\lambda q} dq \tag{10}$$

where $\hat{\rho}$ is the unit vector along ρ. The z-component of the vortex magnetic field at the SC film surface[14]-[15] is

$$B_z^s(\rho) = \frac{\phi_0}{2\pi\lambda^2}\left[\frac{\lambda}{2\rho} - \frac{\pi}{8}\left(H_0(\frac{\rho}{2\lambda}) - N_0(\frac{\rho}{2\lambda})\right)\right] \tag{11}$$

where $H_0(x)$ and $N_0(x)$ are the zero order Struve and Neumann functions respectively[16].

III. THE PINNING POTENTIAL AND ENERGY OF FCD-SV SYSTEM

The interaction between a superconducting vortex and a non magnetic columnar defect was first considered by Mkrtchyan and Schmidt[17] and later in the work of Buzdin *et.al.*[18]-[22]. In these studies, it was shown that the pinning potential U_p created by a non magnetic columnar defect of radius $R > \sqrt{2}\xi$ is

$$U_p(\rho) = \begin{cases} -\epsilon_0 \ln(\frac{R}{\sqrt{2}\xi}), & \rho < R \\ \epsilon_0 \ln\left[1 - \left(\frac{\sqrt{2}R}{\sqrt{2}\rho+\xi}\right)^2\right], & R < \rho < \lambda \end{cases}$$

where ξ is the SC coherence length and $\epsilon_0 = \frac{\phi_0^2}{16\pi^2\lambda}$ is the energy scale of the vortex self interaction.

If the columnar defect is ferromagnetic, then an extra contribution to the pinning would appear due to the interaction between the superconductor and ferromagnet. In the presence of vortices in the superconductor, the total energy of the system is made up of five different contributions and can be written as

$$U = U_{sv} + U_{vv} + U_p + U_{mv} + U_{mm} \tag{12}$$

where U_{sv} is the energy of N non-interacting vortices, U_{vv} is the vortex-vortex interaction, U_{mv} is the interaction energy between the FM and the SC, and U_{mm} is the FM dot self interaction. In[7], it was shown that the total energy of the system can be written as:

$$U = \int \left[\frac{n_s\hbar^2}{8m_e}(\nabla\chi)^2 - \frac{n_s\hbar e}{4m_e c}(\nabla\chi \cdot \mathbf{A}) - \frac{1}{2}\mathbf{M}\cdot\mathbf{B}\right] d^3x \tag{13}$$

where c is the speed of light. The vectorial quantities $\mathbf{A}$, and $\mathbf{B}$ are the total vector potential and magnetic field due to the vortices and the ferromagnetic columnar defect.

In Ref.23, Buzdin considered the problem of vortex craetion in a superconductor penetrated by non-magnetic columnar defects and placed in external magnetic field. He found that at high magnetic field giant vortices are more energy favorable when the columnar defect is rather thick. The case of rather thick defects $R/\lambda \sim 1$ is experimentally more interesting and easier to realize. Marmorkos *et.al.*[8] showed the possibility of formation of a giant vortex with multiple flux quanta around the magnetic dot instead of singly quantized vortices. Therefore, I will assume that $\xi \ll R \le \lambda$ which make the result of Ref.23 regarding the formation of giant vortices applicable to our problem.

The phase gradient of the SC order parameter in the presence of a giant superconducting vortex with vorticity ν is $\nabla\chi = \nu\frac{(\rho-\rho_0)\times\hat{z}}{|\rho-\rho_0|^2}$, where ρ_0 is the location of the vortex. The total energy for a system consisting of a superconducting vortex and a ferromagnetic columnar defect is

$$\begin{aligned} U(\rho_0) &= \nu^2\epsilon_0\ln(\frac{\lambda}{\xi}) + \nu^2 U_p(\rho_0) \\ &\quad -2\nu\epsilon_m\frac{R}{\lambda}\int_0^\infty \frac{J_0(q\rho_0)J_1(qR)(1-e^{-\frac{qL}{2}})}{q(1+2\lambda q)}dq \end{aligned} \tag{14}$$

where $\epsilon_m = M\phi_0\lambda$ and U_{mm} is the self interaction of the FCD is ignored since it does not affect the superconducting state. It is clear that if the magnetization of the columnar defect is large enough so that the interaction term becomes larger than other terms in (14) then the vortex can appear spontaneously in the superconductor. Erdin *et.al.*[7] showed that spontaneous creation of vortices takes place most easily in the proximity of the superconducting transition temperature T_s.

The interaction term in Eq.(14) depends on L. In the limit of $L/R \ll 1$, the interaction term in Eq.(14) reduces to the result obtained for infinitely thin magnetic dot.[7] In the remainder of this work, I will assume $L \to \infty$. Numerical minimization of the total energy U sets $\rho_0 = 0$, hence the vortex center must be on the axis of the FCD.

The energy of a system of a singly quantized vortex $\nu = 1$ coupled to a columnar defect is shown in Fig. 2. The solid line represents the energy of the system when the columnar defect is non-magnetic, while the dashed line shows the energy of the system when the defect is ferromagnetic. Note that the pinning force, $-\nabla U(\rho)$, is not zero for $\rho < R$ in contrast with the case of non-magnetic columnar defect.

For each value of M there is a corresponding critical value of ν which I call it here ν_c. The minimization of $U(\rho_0 = 0)$ with respect to ν yields

$$\nu = \frac{16\pi m_0}{\ln(\frac{\lambda}{R})}\frac{R}{\lambda}\int_0^\infty \frac{J_1(qR)}{q(1+2\lambda q)}dq \tag{15}$$

where $m_0 = M/M_0$ with $M_0 = \phi_0/\pi\lambda^2$. The critical value of ν is the closest integer to the value of ν given by

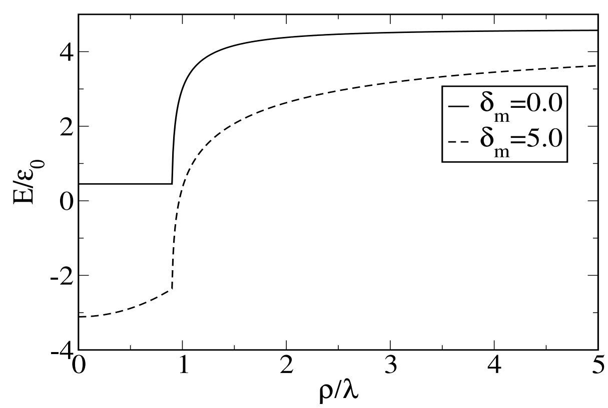

FIG. 2: The total energy of the CD-SV system for $\lambda = 1000$nm, $\xi = 10$nm and the radius $R = 900$nm.. The solid line is for the case when the CD is non-magnetic while the dashed one is for ferromagnetic CD.

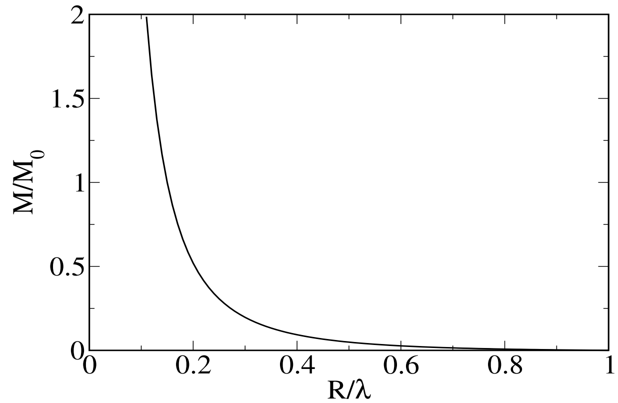

FIG. 4: The curve represents the threshold value of the magnetization of the FCD in units of $M_0 = \phi_0/\pi\lambda^2$. All points above the curve represent a phase with vortices while the region under the curve is vortexless.

Eq.(15). The vorticity of the spontaneously created giant vortex is plotted in Fig. 3 as a function of m_0 and R/λ. Note that at when $R/\lambda \ll 1$ then very large values of m_0 are required for the spontaneous creation of a vortex. Note that for any fixed value of ν_c, Eq.(15) gives us the

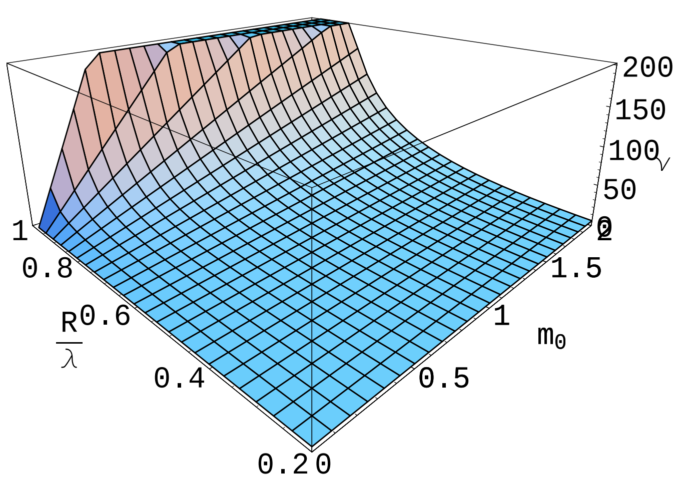

FIG. 3: (Color Online) The plot of ν as a function of m_0 and R/λ.

critical value of the magnetization $M_c(\nu_c)$ at which a vortex with vorticity ν_c appear as a function of the radius R. The dependance of $M_c(\nu_c)$ on R for $\nu_c = 1$ is depicted in Fig. 4. The region under the curve in Fig. 4 represents a vortexless phase while the one above the curve represents phases with vortices.

Note that when the radius of the columnar defect is very close to λ giant vortices with large flux quanta are expected even at not so large values of m_0. This is because when $R \approx \lambda$, the vortex self energy goes to zero while its energy of interaction with the columnar defect is not zero and negative; therefore, giant vortices with large value of ν are energy favorable.

The curve in Fig. 4 is the curve for the first phase transition (No vortex Phase → One vortex phase), it should be emphasized here that there is a discrete set of curves above this curve that refer to a series of phase transitions between phases with different number of flux quanta. The transition from a vortexless phase to a phase with a single vortex for a system in which $R = 50$ nm and $\lambda = 100$ nm occurs at the approximate value of $M_c = 66.85$ G.

IV. AN SC FILM PIERCED BY A SQUARE ARRAY OF FCDS

Now, let us consider a superconducting thin film pierced by a square array of $N \times N$ ferromagnetic columnar defects. Let each FCD be infinitely long ($L \to \infty$) with a radius $\xi \ll R \leq \lambda$ and magnetization $M \geq M_c(\nu_c)$. The magnetization of the array can be written in Fourier space as follows

$$\mathbf{M_K} = 4\pi^2 MR \frac{J_1(qR)}{q} \delta(k_z)\hat{z} \sum_j e^{\imath \mathbf{q}\cdot\boldsymbol{\rho}_\mathbf{j}} \tag{16}$$

where ρ_i is the location of the i-th FM defect and the sum runs over all the lattice sites. The location of the FCD is defined by two integers l and s as $\rho_i = la\hat{x} + sa\hat{y}$ where a is the lattice spacing of the FCD array. Solving Eq.(3) for the magnetization (16) yields

$$\tilde{\mathbf{A}}_K^m = \frac{16\pi^2 \imath MRJ_1(qR)}{q(k_z^2+q^2)}\left(\pi\delta(k_z) - \frac{1}{q(1+2\lambda q)}\right)\sum_j e^{\imath \mathbf{q}\cdot\rho_j} \times(\mathbf{q}\times\hat{z}) \quad (17)$$

Let $\mathbf{Q}$ be a vector of the reciprocal lattice defined as $\mathbf{Q} = n\mathbf{Q}_1 + m\mathbf{Q}_2$ where n and m are two integers with $\mathbf{Q_1} = \frac{2\pi}{a}\hat{x}$ and $\mathbf{Q_2} = \frac{2\pi}{a}\hat{y}$. The z component of the array's magnetic field at $z = 0$ is

$$B_z(x,y,z=0) = 16\pi^2 n_0 M\lambda R\sum_{\mathbf{Q}}\frac{J_1(QR)}{1+2\lambda Q}e^{-\imath\mathbf{Q}\cdot\rho} \quad (18)$$

where $n_0 = 1/a^2$ is the density of columnar defects in the array. The summation over the defect positions is calculated with the help of the identity

$$\sum_i \int f(q) e^{\imath\mathbf{q}\cdot\rho_i}\frac{d^2q}{(2\pi)^2} = \sum_{\mathbf{Q}} n_0 f(\mathbf{Q}) \quad (19)$$

The magnetic flux which penetrate the superconductor must remain constant; therefore, the number of vortices and the number of antivortices that are spontaneously created in the superconductor must be equal. In the absence of external magnetic field, the equilibrium configuration is the one with vortices pinned at the axes of the FCDs and antivortices are located at the centers of the unit cells of the FCD lattice. The square lattice of antivortices is identical to that of the vortices with a shift by $\mathbf{a}/2 = (a/2, a/2)$. Note that in the case of a single columnar defect, the antivortex is repelled (pushed far away) from the FCD; therefore, it was not taken into account in the calculations. However, in the case when the SC is penetrated by an array of FCD in the absence of external magnetic field, one must take antivortices into account on equal footing with vortices. In equilibrium, the force acting on any flux line (FL) in the superconductor is zero. Pinning forces appear as soon as this equilibrium is disturbed e.g. by passing an electric current to the superconducting film. The remaining part of this section will focus on the problem of calculating these forces. The vortex and antivortex lattices are both regular and periodic. Therefore, if a small current is applied to the superconductor then all vortices will move together in one direction and all antivortices will move together in the opposite direction. Let us assume that the vortex lattice is displaced with respect to its equilibrium position by a small amount $\Delta\rho = (x, y)$ then the lattice of antivortices will be displaced by an amount $-\Delta\rho$. Considering the case when M is just above $M_c(\nu_c = 1)$. The flux line self energy and the FL-FL interaction energy together can be written using Eq.(13) as

$$E_{vv} = \frac{\mathcal{A}n_0^2\phi_0^2}{2\pi}\sum_{\mathbf{Q}}\frac{1}{Q(1+2\lambda Q)}\left[1-\cos(\mathbf{Q}\cdot(\frac{\mathbf{a}}{2}-2\Delta\rho))\right] \quad (20)$$

where $\mathcal{A}$ is the area of the SC film. The energy of interaction between the flux lines and the ferromagnetic array is

$$E_{mv} = -2\pi\mathcal{A}n_0^2 MR\lambda\sum_{\mathbf{Q}}\frac{J_1(QR)}{Q^2(1+2\lambda Q)}\left[\cos(\mathbf{Q}\cdot\Delta\rho) - \cos(\mathbf{Q}\cdot(\frac{\mathbf{a}}{2}-\Delta\rho)\right] \quad (21)$$

To simplify the calculations, I use

$$\sum_{\mathbf{Q}} \Rightarrow \mathcal{A}_0\int\frac{d^2Q}{(2\pi)^2} \quad (22)$$

where $\mathcal{A}_0 = a^2$ is the area of the array's unit cell. Therefore, I find the energy per flux line is now dependent on the displacement $\Delta\rho$ as follows

$$\mathcal{E}_o = \frac{\phi_0^2}{8\pi^2}\int_0^\infty\frac{\left[1-J_0(q|\frac{\mathbf{a}}{2}-2\Delta\rho|)\right]}{1+2\lambda q}dq - M\phi_0 R\int_0^\infty\frac{J_1(qR)\left[J_0(q|\Delta\rho|)-J_0(q|\frac{\mathbf{a}}{2}-\Delta\rho|)\right]}{q(1+2\lambda q)}dq \quad (23)$$

Note that the number of FCD is N^2 while the number of flux lines is $2N^2$. The pinning force acting on any vortex or antivortex can now be calculated using $\mathbf{f}_p = -\nabla U(\Delta\rho)$ where $U(\Delta\rho) = \mathcal{E}_o - \mathcal{E}_e$ with $\mathcal{E}_e$ is a constant equals to the energy of the system in equilibrium. Note that I did not include the term due to the Mkrtchyan and Schmidt in the result I obtained in Eq.(24) because it is rather straightforward to calculate its effect.

V. CONCLUSION

In conclusion, I studied the interaction between superconducting vortices in a thin film pierced by a ferromagnetic columnar defect. I calculated the magnetic field of the FCD in the presence of the SC and the distribution of screening currents in the superconductor. If the magnetization of the FCD exceeds a critical value, then the

FCD interaction with the vortex will overcome the vortex self energy leading to the spontaneous creation of vortices in the superconductor. I showed that vortex pinning is strongly enhanced due to the contribution from the interaction between the magnetic field of vortices and the magnetization of the FCD outside the plane of the SC and its close proximity. This extra contribution to vortex pinning is a major difference between this study and other studies in the literature which focused on dipole-like or two-dimensional ferromagnetic structures. These results were generalized to include the case when the SC film is pierced by an array of FCD. A detailed analysis of the dynamical properties of this system will be reported elsewhere.

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